AA - Laborator 4

1. Rezolvati folosind metoda Master:

a) $T(n)=16*T(n/4)+n^2$

$a=16, b=4 \Rightarrow n^{\log_b a}=n^2$
$f(n)=n^2$

$\Rightarrow f(n)=n^2=\Theta(n^2)=\Theta(n^{\log_b a}) \Rightarrow T(n)=\Theta(n^{\log_b a}*lg\, n)=\Theta(n^2*lg\, n)\ \ (caz\, 2)$

b) $T(n)=4*T(n/2)+\Theta(n)$

$a=4, b=2 \Rightarrow n^{\log_b a}=n^2$
$f(n)=k*n,\ \ k\ \ constant$

$\Rightarrow f(n)=k*n=O(n)=O(n^{2-1})=O(n^{\log_b a-1}) \Rightarrow \epsilon=1$
$\Rightarrow T(n)=\Theta(n^{\log_b a})=\Theta(n^2)\ \ (caz\, 1)$

c) $T(n)=2*T(n/2)+\Theta(n^3)$

$a=2, b=2 \Rightarrow n^{\log_b a}=n$
$f(n)=k*n^3,\ \ k\ \ constant$

c1) $f(n)=k*n^3=\Omega(n^3)=\Omega(n^{1+2})=\Omega(n^{\log_b a+2}) \Rightarrow \epsilon=2$

c2) $2*f(n/2) \leqslant c*f(n),\ \ \forall\, n \geqslant n_0$
$2*k*n^3/2^3 \leqslant c*k*n^3,\ \ \forall\, n \geqslant n_0$
$(1/4) \leqslant c,\ \ \forall\, n \geqslant n_0$

Alegem $c=1/4,\ n_0=1$.

Rezulta din cazul 3: $T(n)=\Theta(k*n^3)=\Theta(n^3)$.

d) $T(n)=2*T(n/2)+\Theta(n\, lg\, n)$

$a=2, b=2 \Rightarrow n^{\log_b a}=n$
$f(n)=k*n\, lg\, n$

caz 2: $k*n\, lg\, n \neq \Theta(n)$
caz 1: $k*n\, lg\, n \neq O(n^{1-\epsilon}),\ \ \epsilon>0$
caz 3: Aratam ca $k*n\, lg\, n \neq \Omega(n^{1+\epsilon}),\ \ \epsilon>0$

$k*n\lg n=\Omega(n^{1+\epsilon})\Rightarrow$

$\exists c\in\mathbb{R}_+^*,\ \exists n_0\in\mathbb{N}^*\ \ ai\ \ c*n^{1+\epsilon}\leqslant k*n\lg n,\ \ \forall n\geqslant n_0$

$\Rightarrow c*n^{\epsilon}\leqslant k*\lg n,\ \ \forall n\geqslant n_0\ \Rightarrow\ c\leqslant(k*\lg n)/n^{\epsilon},\ \ \forall n\geqslant n_0$

Pentru $n\to\infty$ rezulta ca $c\leqslant 0$ ceea ce reprezinta o contradictie cu faptul ca $c\in\mathbb{R}_+^*$.

Rezulta ca nu se poate aplica metoda Master.

2. a) $T(n)=2*T(n/2)+\Theta(n\lg n),\quad T(1)=\Theta(1)$

$T(n)=2*T(n/2)+\Theta(n\lg n)\qquad *2^0$

$T(n/2)=2*T(n/2^2)+\Theta((n/2)\lg(n/2))\qquad *2^1$

...

$T(n/2^k)=2*T(n/2^{k+1})+\Theta((n/2^k)\lg(n/2^k))\qquad *2^k$

Adunam relatiile de mai sus, consideram si faptul ca $n=2^{k+1}$ si avem:

$$T(n)=2^{k+1}*T(1)+\sum_{i=0}^{k}2^i*\Theta((n/2^i)\lg(n/2^i))=n*\Theta(1)+\Theta(\sum_{i=0}^{k}n*(\lg n-i))=$$
$$=\Theta(n)+\Theta(\sum_{i=0}^{k}n\lg n-\sum_{i=0}^{k}n*i)=\Theta(n)+\Theta((k+1)*n\lg n-n*\sum_{i=1}^{k}i)=$$
$$=\Theta(n+n*\lg^2 n-n*k(k+1)/2)=\Theta(n+n*\lg^2 n-(n/2)*(\lg n-1)*\lg n)=$$
$$=\Theta(n+n*\lg^2 n-(n*\lg^2 n)/2+(n*\lg n)/2)=\Theta(n*\lg^2 n)$$

b) $T(n)=2*T(n-1)+\Theta(1),\quad T(1)=\Theta(1)$

$T(n)=2*T(n-1)+\Theta(1)\qquad *2^0$

$T(n-1)=2*T(n-2)+\Theta(1)\qquad *2^1$

...

$T(2)=2*T(1)+\Theta(1)\qquad *2^{n-2}$

Adunam relatiile de mai sus si obtinem:

$$T(n)=2^{n-1}*T(1)+\sum_{i=0}^{n-2} 2^i*\Theta(1)=2^{n-1}*\Theta(1)+\Theta(\sum_{i=0}^{n-2} 2^i)=\Theta(2^{n-1})+\Theta((2^{n-1}-1)/(2-1))=$$
$$=\Theta(2^{n-1}+2^{n-1}-1)=\Theta(2^n-1)=\Theta(2^n)$$

3. Rezolvati recurenta: $T(n)=2*T(\sqrt{n})+lg\, n$.

Fie $n=2^m$

$$T(2^m)=2*T(2^{m/2})+lg\, 2^m$$

$$T(2^m)=2*T(2^{m/2})+m$$

Fie $S(m)=T(2^m)$. Atunci:

$$S(m)=2*S(m/2)+m$$

Folosim metoda Master pentru a rezolva aceasta recurenta:

$$a=2,b=2 \Rightarrow m^{\log_b a}=m$$

$$f(m)=m=\Theta(m)=\Theta(m^{\log_b a})$$

Din cazul 2 avem:

$$S(m)=\Theta(m^{\log_b a}*lg\, m)=\Theta(m*lg\, m)$$

$$T(2^m)=\Theta(m*lg\, m)$$

$$n=2^m \Rightarrow m=lg\, n$$

Rezulta ca: $T(n)=\Theta(lg\, n*lg\, lg\, n)$.