Prolog Search

Implementing Search in Prolog

- How to represent the problem
- Uninformed Search
  - depth first
  - breadth first
  - iterative deepening search
- Informed Search
  - Hill climbing
  - Graph Search
    - which can do depth first, breadth first, best first, Algorithm A, Algorithm A*, etc.

Representing the Problem

- Represent the problem space in terms of these predicates:
  - goal/1
  - start/1
  - arc/3, arc/2
- **goal(S)** is true iff S is a goal state.
- **start(S)** is true iff S is an initial state.
- **arc(S1,S2,N)** is true iff there is an operator of cost N that will take us from state S1 to state S2.
- arc(S1,S2) :- arc(S1,S2,_)

Sam Lloyd's 15 puzzle

- The eight puzzle is a reduced version of the "fifteen puzzle" which is often attributed to the famous American game and puzzle designer Sam Loyd (1841-1911)
  He invented the modern version of Parcheesi.
- He didn't invent it, but he did invent a variation where the 114 and 15 tiles are swapped, with all other tiles in place.
- In the 1870's he offered a $1000 prize for whoever could solve this.
- He knew however that this puzzle could not possibly be solved.
- The 20,000,000,000,000 states form two disconnected semi-spaces
- He made a lot of money on this puzzle.

Eight Puzzle Example

- Represent a state as a list of the eight tiles and o for blank.
- E.g., [1,2,3,4,o,5,6,7,8] for


```
goal([1,2,3,
      4,o,5,
      6,7,8]).
arc([o,B,C,
     D,E,F,
     G,H,I],
    [B,o,C,
     D,E,F,
     G,H,I]).
```

Missionaries and Cannibals Solution

		Far side		Near side
0	Initial setup:	MMMCCC	B	-
1	Two cannibals cross over:	MMMC	B	CC
2	One comes back:	MMMCC	B	C
3	Two cannibals go over again:	MMM	B	CCC
4	One comes back:	MMMC	B	CC
5	Two missionaries cross:	MC	B	MMCC
6	A missionary & cannibal return:	MMCC	B	MC
7	Two missionaries cross again:	CC	B	MMMC
8	A cannibal returns:	CCC	B	MMM
9	Two cannibals cross:	C	B	MMMCC
10	One returns:	CC	B	MMMC
11	And brings over the third:	-	B	MMMCCC

Missionaries and Cannibals

```
% Represent a state as
% [ML,CL,MR,CL,B]
start([3,3,0,0,left]).
goal([0,0,3,3,X]).

% eight possible moves…
arc([ML,CL,MR,CR,left],
    [ML2,CL,MR2,CR,right]):-
  % two Ms row right
  MR2 is MR+2,
  ML2 is ML-2,
  legal(ML2,CL2,MR2,CR2).

arc([ML,CL,MR,CR,left],
    [ML2,CL,MR2,CR,right]):-
  % one M & one C row right
  MR2 is MR+1,
  ML2 is ML-1,
  CR2 is CR+1,
  CL2 is CL-1,
  legal(ML2,CL2,MR2,CR2).

legal(ML,CL,MR,CL) :-
  % is this state a legal one?
  ML>0, CL>0, MR>0, CL>0,
  ML>=CL, MR>=CR.
```

Depth First Search (1)

```
%% this is surely the simplest possible DFS.
dfs(S,[S]) :- goal(S).
dfs(S,[S|Rest]) :-
    arc(S,S2),
    dfs(S2,Rest).
```

Depth First Search (2)

```
%% this is surely the simplest possible DFS:-
:- ensure_loaded(showPath).
dfs :- dfs(Path), showPath(Path).
dfs(Path) :- start(S), dfs(S,Path).
dfs(S,[S]) :- goal(S).
dfs(S,[S|Rest]) :-
    arc(S,S2),
    dfs(S2,Rest).
```

Showpath

```
:- use_module(library(lists)).

%% Print a search path
showPath(Path) :-
  Path=[First|_],
  last(Path,Last),
  nl, write('A solution from '),
  showState(First),
  write(' to '),
  showState(Last),
  nl,
  foreach1(member(S,Path),(write('  '), showState(S), nl)).

% call Action for each way to prove P.
foreach1(P,Action) :- P,once(Action),fail.
foreach1(_,_).

%% once(P) execute's P just once.
once(P) :- call(P), !.

showState(S) :- writeState(S) -> true| write(S).
```

Depth First Search which avoids loops

```
/* this version of DFS avoids loops by keeping track of the path
   as it explores.  It's also tail recursive! */
:- ensure_loaded(library(lists)).

dfs(S,Path) :-
   dfs1(S,[S],ThePath),
   reverse(ThePath,Path).

dfs1(S,Path,Path) :- goal(S).
dfs1(S,SoFar,Path) :-
  arc(S,S2),
  \+(member(S2,SoFar)),
  dfs1(S2,[S2|SoFar], Path).
```

Breadth First Search

```
bfs :- start(S), bfs(S).

bfs(S) :-
   empty_queue(Q1),
   queue_head(S,Q1,Q2),
   bfs1(Q2).

bfs1(Q) :-
   queue_head(S,_,Q),
   arc(S,G),
   goal(G).

bfs1(Q1) :-
   queue_head(S,Q2,Q1),
   findall(X,arc(S,X), Xs),
   queue_last_list(Xs,Q2,Q3),
   bfs1(Q3).
```

Breadth First Search

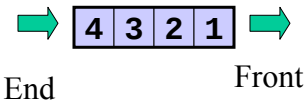
```
:- use_module(library(queues)).

bfs(S,Path) :-
   empty_queue(Q1),
   queue_head([S],Q1,Q2),
   bfs1(Q2,Path).

bfs1(Q,[G,S|Tail]) :-
   queue_head([S|Tail],_,Q),
   arc(S,G),
   goal(G).

bfs1(Q1,Solution) :-
   queue_head([S|Tail],Q2,Q1),
   findall([Succ,S|Tail],
        (arc(S,Succ), \+member(Succ,Tail)),
        NewPaths),
   queue_last_list(NewPaths,Q2,Q3),
   bfs1(Q3,Solution).
```

Note on Queues



- :- use_module(library(queues))
- **empty_queue(?Q)**
  - Is true if Q is a queue with no elements.
- **queue_head(?Head, ?Q1, ?Q2)**
  - Q1 and Q2 are the same queues except that Q2 has Head inserted in the front. Can be used to insert or delete from the head of a Queue.
- **queue_last(?Last, ?Q1, ?Q2)**
  - Q2 is like Q1 but have Last as the last element in the queue. Can be used to insert or delete from the end of a Queue.
- **list _ queue(+List, ?Q)**
  - Q is the queue representation of the elements in list List.
- Note: Queues are represented as a pair (L,Hole) where list L ends with a variable unified with Hole.

More on Queues

```
enqueue(X,Qin,Qout) :-
  queue_last(X,Qin,Qout).

dequeue(X,Qin,Qout) :-
  queue_head(X,Qout,Qin).
```

Iterative Deepening

```
id(S,Path) :-
   from(Limit,1,5),
   id1(S,0,Limit,Path).

id1(S,Depth,Limit,[S]) :-
   Depth<Limit,
   goal(S).

id1(S,Depth,Limit,[S|Rest]) :-
   Depth<Limit,
   Depth2 is Depth+1,
   arc(S,S2),
   id1(S2,Depth2,Limit,Rest).
```

```
% from(-Var,+Val,+Inc)
% instantiates Var to #s
% beginning with Val &
% incrementing by Inc.

from(X,X,Inc).
from(X,N,Inc) :-
  N2 is N+Inc,
  from(X,N2,Inc).
```

```
| ?- from(X,0,5).
X = 0 ? ;
X = 5 ? ;
X = 10 ? ;
X = 15 ? ;
X = 20 ? ;
X = 25 ? ; ...
yes
| ?-
```

Informed Search

- For informed searching we'll assume a heuristic function *h(+S,?D)* that relates a state to an estimate of the distance to a goal.
- Hill climbing
- Best first search
- General graph search which can be used for
  - depth first search
  - breadth first search
  - best first search
  - Algorithm A
  - Algorithm A*

Hill Climbing

```
hc(Path) :- start(S), hc(S,Path).

hc(S,[S]) :- goal(S), !.

hc(S,[S|Path]) :-
  h(S,H),
  findall(HSS-SS,
          (arc(S,SS),h(SS,Dist)),
           L),
  keysort(L,[BestD-BestSS|_]),
  H>BestD -> hc(BestSS,Path)
           ;   (dbug("Local max:~p~n", [S]), fail).
```

Best First Search

```
:- ensure_loaded(showPath).
:- ensure_loaded(dbug).
:- use_module(library(lists)).

/* best first search is like dfs but we chose as the next node to expand  the one that
   seems closest to the goal using the heuristic function h(?S,-D) */

bestfs :- bestfs(Path), showPath(Path).

bestfs(Path) :- start(S), bestfs(S,Path).

bestfs(S,[S]) :- goal(S), !.

bestfs(S,[S|Path]) :-
  findall(Dist-SS,
        (arc(S,SS), h(SS,Dist)),
        L),
  keysort(L,SortedL),
  member(_-NextS,SortedL),
  bestfs(NextS,Path).
```

Graph Search

The graph is represented by a collections of facts of the form:
node(S,Parent,Arcs,G,H) where

- **S** is a term representing a state in the graph.
- **Parent** is a term representing S's immediate parent on the best known path from an initial state to S.
- **Arcs** is either *nil* (no arcs recorded, i.e. S is in the set open) or a list of terms *C-S2* which represents an arc from S to S2 of cost C.
- **G** is the cost of the best known path from the state state to S.
- **H** is the heuristic estimate of the cost of the best path from S to the nearest goal state.

Graph Search

In order to use gs, you must define the following predicates:

- **goal(S)** true if S is a term which represents the goal state.
- **arc(S1,S2,C)** true iff there is an arc from state S1 to S2 with cost C.
- **h(S,H)** is the heuristic function as defined above.
- **f(G,H,F)** F is the meteric used to select which nodes to expand next. G and H are as defined above. Default is *"f(G,H,F) :- F is G+H."*.
- **start(S)** (optional) S isthe state to start searching from.

```
gs(Start,Solution) :-
  retractall(node(_,_,_,_,_)),
  addState(Start,Start,0,0),
  gSearch(Path),
  reverse(Path,Solution).

gSearch(Solution) :-
  select(State),
  (goal(State)
   -> collect_path(State,Solution)
      |(expand(State),gSearch(Solution))).

select(State) :-
  % find open state with minimal F value.
  findall(F-S,
   (node(S,P,nil,G,H),f(G,H,F)), OpenList),
  keysort(OpenList,[X-State|Rest]).

expand(State) :-
  dbug("Expanding state ~p.~n",[State]),
  retract(node(State,Parent,nil,G,H)),
  findall(ArcCost-Kid,
    (arc(State,Kid,ArcCost),
     add_arc(State,Kid,G,ArcCost)),
    Arcs),
   assert(node(State,Parent,Arcs,G,H)).
```

```
add_arc(Parent,Child,ParentG,ArcCost) :-
  % Child is a new state, add to the graph.
  (\+node(Child,_,_,_,_)),
  G is ParentG+ArcCost,
  h(Child,H),
  dbug("Adding state ~p with parent ~p and
   cost ~p.~n",[Parent,Child,G]),
  assert(node(Child,Parent,nil,G,H)), !.

add_arc(Parent,Child,ParentG,ArcCost) :-
  % Child state is already in the graph.
  % update cost if the new path better.
  node(Child,_CurrentParent,Arcs,CurrentG,H),
  NewG is ParentG+ArcCost,
  CurrentG>NewG, !,
  dbug("Updating ~p 's cost thru ~p to
   ~p.~n",[State,Parent,NewG]),
  retract(node(Child,_,_,_,_)),
  assert(node(Child,Parent,Arcs,NewG,H)),
  % better way to get to any grandKids?
  foreach(member(ArcCost-Child,Arcs),
    (NewCostToChild is NewG+ArcCost,
    update(Child,State,NewCostToChild))).

add_arc(_,_,_,_).

collect_path(Start,[Start]) :-
  node(Start,Start,_Arcs,0,_H).
collect_path(S,[S|Path]) :-
  node(S,Parent,_,_,_),
  collect_path(Parent,Path).
```

Note on Sorting

- sort(+L1,?L2)
  - Elements of the list L1 are sorted into the standard order and identical elements are merged, yielding the list L2.

    | ?- sort([f,s,foo(2),3,1],L).

    L = [1,3,f,s,foo(2)] ?
- keysort(+L1,?L2)
  - List L1 must consist of items of the form *Key-Value*. These items are sorted into order w.r.t. Key, yielding the list L2. No merging takes place.

    | ?- keysort([3-bob,9-mary,4-alex,1-sue],L).

    L = [1-sue,3-bob,4-alex,9-mary] ?
  - Example:

    youngestPerson(P) :-

    findall(Age-Person,(person(Person),age(Person,Age)),L),

    keysort(L,[_-P|_].