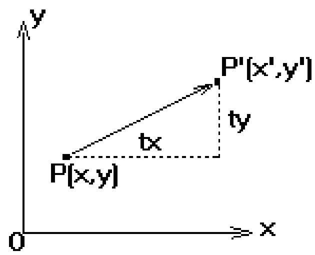
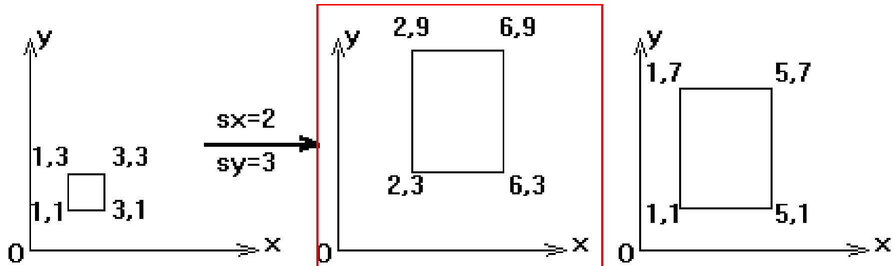
TRANSFORMARI GEOMETRICE 2D (BIDIMENSIONALE)

1. TRANSFORMARILE ELEMENTARE



$$x' = x + t_x$$
$$y' = y + t_y$$

Translatia



Scalarea

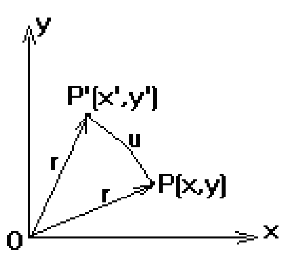
$$[x' \; y'] = [x \; y] \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

Fata de origine

$$x' = x * s_x$$
$$y' = y * s_y$$

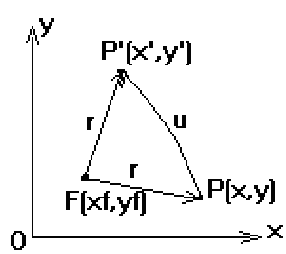
Fata de varful (xf=1, yf=1)

$$x' = x * s_x + x_f - x_f * s_x$$
$$y' = y * s_y + y_f - y_f * s_y$$



Rotatia fata de origine

$$x' = x * \cos(u) - y * \sin(u))$$
$$y' = x * \sin(u) + y * \cos(u))$$

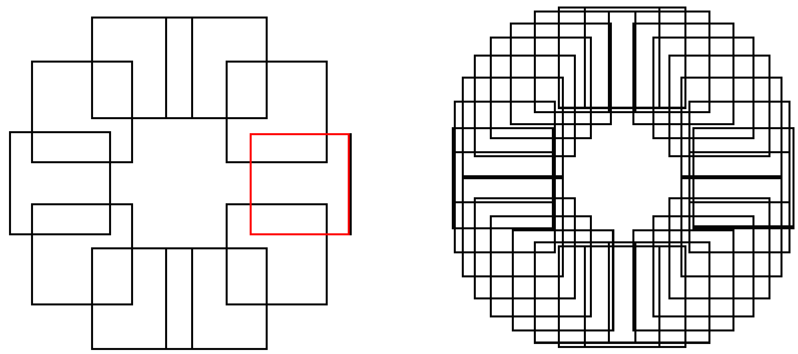


Rotatia fata de un punct oarecare

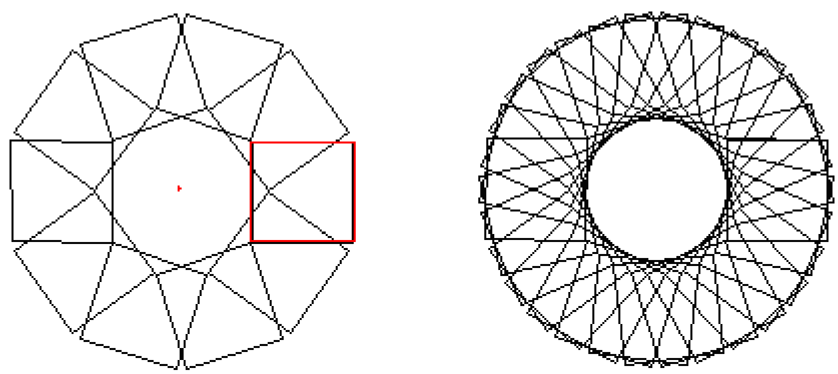
$$x' = x * \cos(u) - y * \sin(u) + x_f - x_f * \cos(u) + y_f * \sin(u)$$
$$y' = x * \sin(u) + y * \cos(u) + y_f - x_f * \sin(u) - y_f * \cos(u)$$

$$[x' \; y'] = [x \; y] \begin{bmatrix} \cos(u) & \sin(u) \\ -\sin(u) & \cos(u) \end{bmatrix}$$

Rotatia fata de origine



Translatia patratului pe circumferinta unui cerc



Rotatia patratului in jurul unui punct

2. COMPUNEREA TRANSFORMARILOR ELEMENTARE

- Inmultirea matricilor corespunzatoare transformarilor
- Coordonate omogene:

P(x,y) → [xw, yw, w] sau

$$\begin{bmatrix} xw \\ yw \\ w \end{bmatrix}$$

xw = x * w; yw = y * w; w – orice numar real

Ex: P(2, 0.5) → [2, 0.5, 1], [4, 1, 2], [20, 5, 10]

Exprimarea transformarilor elementare in coordonate omogene

Translația

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix} \quad \text{sau} \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scalarea față de origine

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{sau} \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotația fața de origine

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} \cos(u) & \sin(u) & 0 \\ -\sin(u) & \cos(u) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{sau} \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(u) & -\sin(u) & 0 \\ \sin(u) & \cos(u) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Expresiile matematice ale scalării şi rotației față de un punct oarecare din plan se pot obține prin compunerea următoarelor transformări:

1. Translația prin care punctul fix al transformării ajunge în origine;
2. Scalarea / rotația față de origine;
3. Translația inversă celei de la punctul 1.

Scalarea față de punctul F (xf, yf)

$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_f & -y_f & 1 \end{bmatrix}\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_f & y_f & 1 \end{bmatrix} \text{ sau}$$

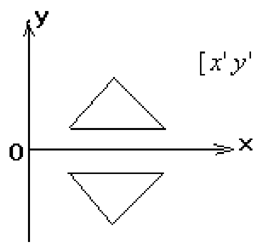
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

3. ALTE TRANSFORMARI 2D

OGLINDIREA

$x' = x$

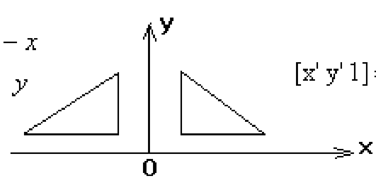
$y' = -y$



$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ sau } \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$x' = -x$

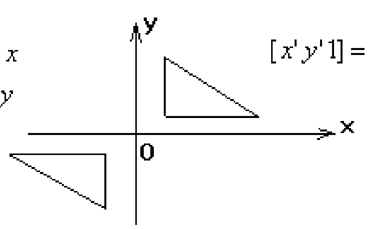
$y' = y$



$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ sau } \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$x' = -x$

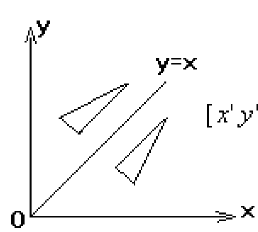
$y' = -y$



$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ sau } \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$x' = y$

$y' = x$



$$[x'\ y'\ 1] = [x\ y\ 1]\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ sau } \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

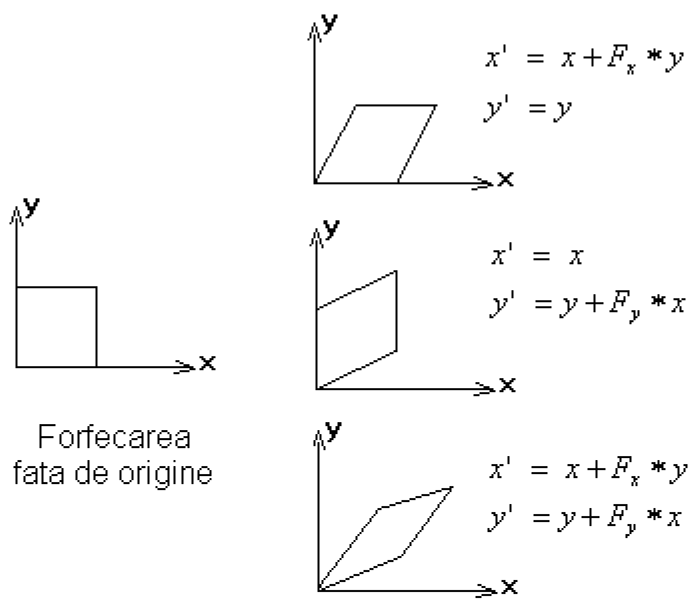
Oglindirea față de o dreaptă oarecare: transformare compusa

1. O translație, astfel încât dreapta su treaca prin origine;
2. O rotație față de origine astfel încât dreapta să se suprapună peste una dintre axele principale;
3. Oglindirea față de axa principală peste care a fost suprapusă dreapta.
4. Rotația inversă celei de la punctul 2;
5. Translația inversă celei de la punctul 1.

În notație matricială, secvența de mai sus se exprimă astfel:

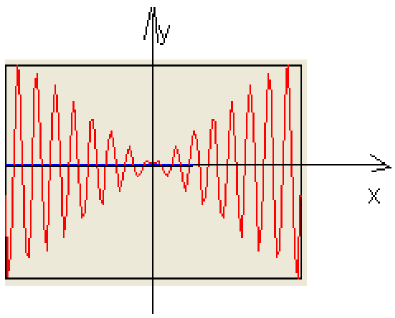
$M = T * R * O * R^{-1} * T^{-1}$ sau $M = T^{-1} * R^{-1} * O * R * T$

FORFECAREA

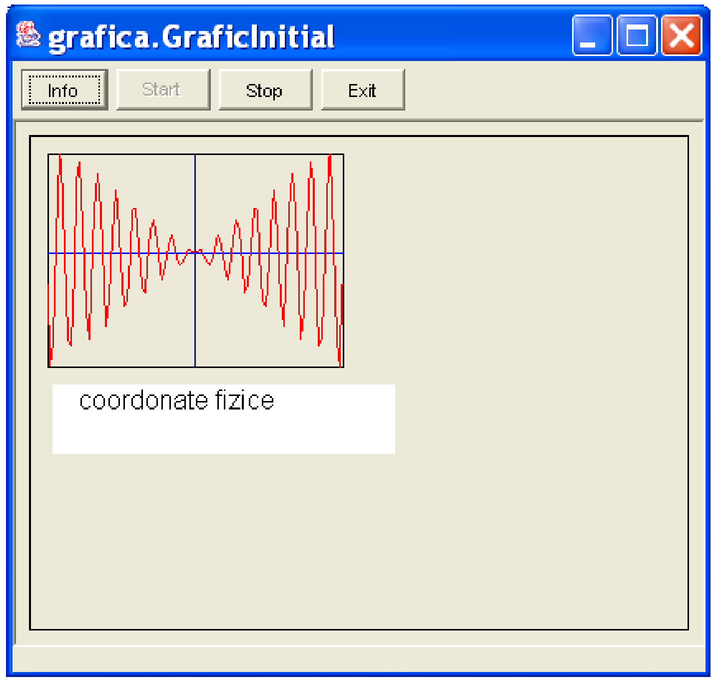


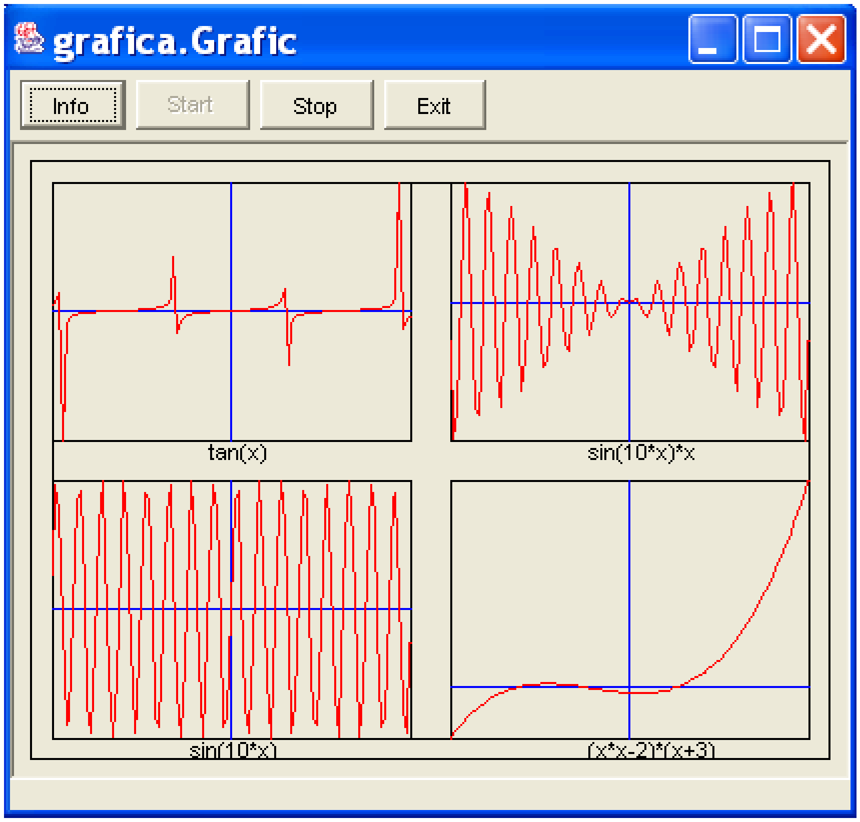
Forfecarea
fata de origine

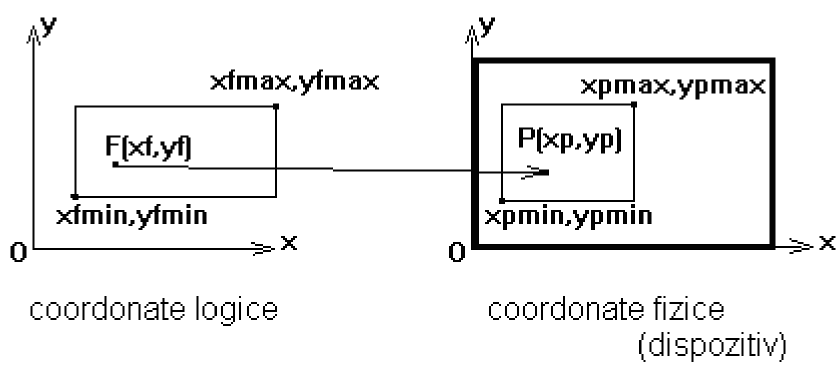
TRANSFORMAREA DE VIZUALIZARE 2D



coordonate logice







$$\frac{xp - xp\min}{xp\max - xp\min} = \frac{xf - xf\min}{xf\max - xf\min}$$

$$\frac{yp - yp\min}{yp\max - yp\min} = \frac{yf - yf\min}{yf\max - yf\min}$$

$$sx = \frac{xp\max - xp\min}{xf\max - xf\min}$$

$$sy = \frac{yp\max - yp\min}{yf\max - yf\min}$$

şi tx = xpmin - sx*xfmin ty = ypmin - sy*yfmin

$$xp = xf * sx + tx$$
$$yp = yf * sy + ty$$