Pentru SLD cu $H(z) = \dfrac{z+5}{z^2+z+1}$ să se traseze caracteristica hodograf.

Rez. prob. se (poate) tratează direct în Z cu DNYQ

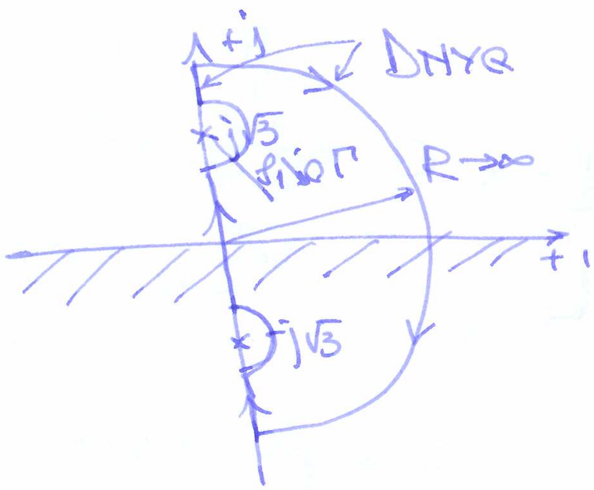
De aceea, se aplică transf. omogr. w

$$H(w) = \frac{\frac{1+w}{1-w}+5}{\frac{(1+w)^2}{(1-w)^2}+\frac{1+w}{1-w}+1} =$$

$$= \frac{(1-w)(-4w+6)}{\cancel{w^2}+\cancel{2w}+1+1-\cancel{w^2}+1+w^2-\cancel{2w}} = \frac{-2(1-w)(2w-3)}{w^2+3}$$

$$= \frac{2(1-w)(-2w+3)}{w^2+3}$$ pt asta, e ca în cazul N

polii sunt: $p_{1,2} = \pm j\sqrt{3}$




simetria → se face studiul doar pe semip. superior

a) axa imaginară: $w = j\eta \quad \eta \in [0,+\infty)\setminus\{\sqrt{3}\}$

$$H(j\eta) = \frac{2(3-j\cdot 2\eta)(1-j\eta)}{3-\eta^2}$$

$$U(\eta) = \frac{2(3-2\eta^2)}{3-\eta^2} \qquad V(\eta) = \frac{-10\eta}{3-\eta^2}$$

| $\eta$ | $0$ | $\sqrt{\frac{3}{2}}$ | $\sqrt{3}$ | $+\infty$ |
|---|---|---|---|---|
| $U(\eta)$ | $2$ | $0$ | $-\infty$ \| $+\infty$ | $4$ |
| $V(\eta)$ | $0$ | $-10\sqrt{\frac{2}{3}}$ | $-\infty$ \| $+\infty$ | $0+$ |

↑ asimptota    ↑ diferență H și Δ semnificative

$U(\eta)=0 \rightarrow \eta=\pm\sqrt{\frac{3}{2}}$

$V\left(\frac{\sqrt{3}}{2}\right)=\frac{-10\sqrt{\frac{3}{2}}}{3-\frac{3}{2}}$

b) $\gamma_1: \; w=j\sqrt{3}+\rho_1 e^{j\alpha}$

$\rho_1 \searrow 0 \quad \alpha\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$H(w)\Big|_{w\in\gamma_1}=$

$$=\frac{2(-j2\sqrt{3}-\cancel{2\rho_1 e^{j\alpha}}+3)(1-j\sqrt{3}-\cancel{\rho_1 e^{j\alpha}})}{\cancel{-\sqrt{3}^2}+j2\sqrt{3}\rho_1 e^{j\alpha}+\cancel{\rho_1^2 e^{j2\alpha}}+\cancel{3}}\Bigg|_{\rho_1\searrow 0}\simeq$$

$$\frac{2(3-j2\sqrt{3})(1-j\sqrt{3})}{j2\sqrt{3}\rho_1 e^{j\alpha}}=Me^{j\beta}$$

$$M=\frac{2\sqrt{21}\cdot\sqrt{4}}{2\sqrt{3}\rho_1}\Bigg|_{\rho_1\searrow 0}\rightarrow\infty$$

$$\beta=-\operatorname{arctg}\frac{2\sqrt{3}}{3}-\operatorname{arctg}\sqrt{3}-\frac{\pi}{2}-\alpha\in\left[-\underbrace{\operatorname{arctg}\frac{2}{\sqrt{3}}}_{\sim\frac{\pi}{4}}-\underbrace{\operatorname{arctg}\sqrt{3}}_{\pi/3},\;-\operatorname{arctg}\frac{2}{\sqrt{3}}-\operatorname{arctg}\sqrt{3}-\pi\right]$$

$\Delta\beta=-\pi$    ! completare la a)

$\eta\rightarrow\sqrt{3}$ pt că polii simpli, hodograful are o asimpt. oblică, de forma

$$m=\lim_{\eta\to\sqrt{3}}\frac{V(\eta)}{U(\eta)}=\lim_{\eta\to\sqrt{3}}\frac{-10\eta}{2(3-2\eta^2)}=\frac{-10\sqrt{3}}{2(-3)}=\frac{5}{\sqrt{3}}$$

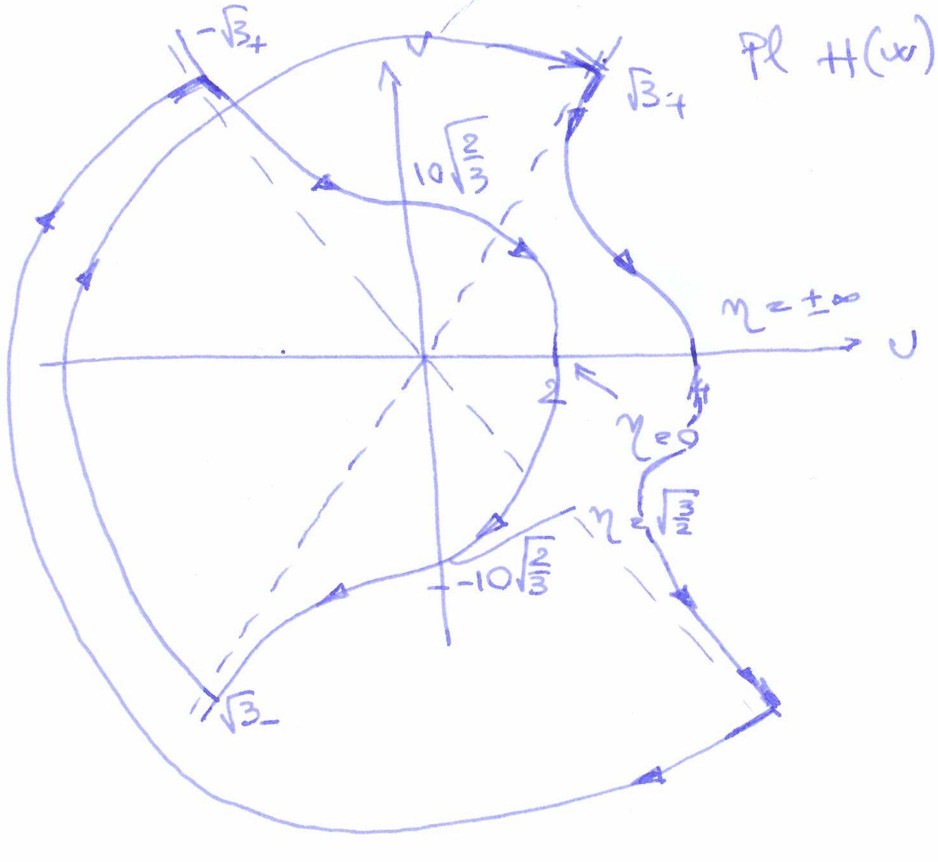
$$n = \lim_{\eta \to \sqrt{3}} \left[ V(\eta) - m U(\eta) \right] = \ldots$$ mai puțin important

c) conturul de la ∞

$w = R e^{j\theta}$ $\quad R \to \infty$

$\theta \in \left[ \frac{\pi}{2}, -\frac{\pi}{2} \right]$

$$H(w) \Big|_{w \in \Gamma} = \frac{2(\cancel{3} - 2Re^{j\theta})(\cancel{1} - Re^{j\theta})}{\cancel{3} + R^2 e^{j2\theta}} \Bigg|_{R \to \infty} \simeq 4$$




Pl $H(w)$

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