Seminar 9 – TS

Trasarea caracteristicii hodograf / loc de transfer / loc Nyquist

<u>Aplicație</u>

Fie SLN : $H(s) = \dfrac{2(-s+2)}{(s^2+1)(s+0.5)(s+1)}$

Să se traseze hodograful pentru SLN-ul dat.

<u>Rezolvare</u>

Pas 1: Se determină polii funcției de transfer:

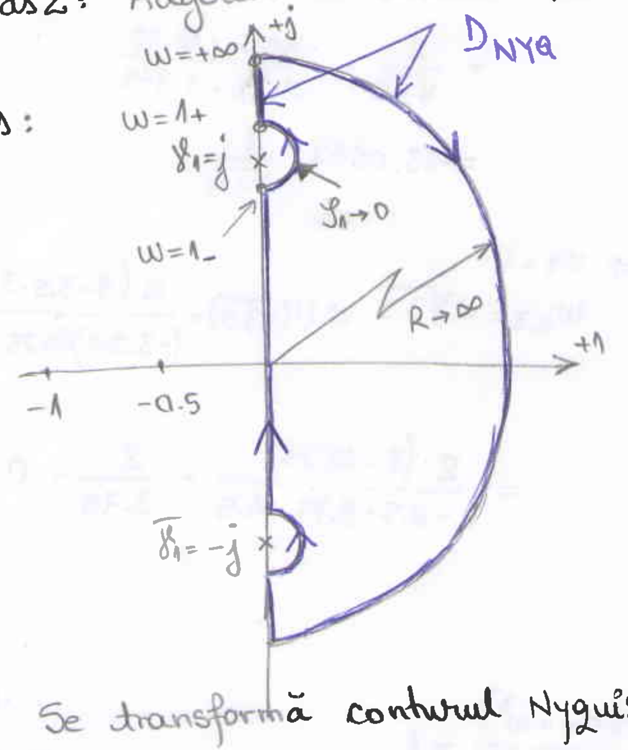
$p_{1,2} = \pm j$

$p_3 = -0.5$

$p_4 = -1$

Pas 2: Alegerea conturului Nyquist : Sensul este dat de creșterea lui ω

Planul s:



Se transformă conturul Nyquist pe porțiuni
- a) axa imaginară
- b) în jurul polilor
- c) la infinit

a) axa imaginară

$s = j\omega$, $\omega \in [0,+\infty) - \{1\}$

$H(j\omega) = U(\omega) + jV(\omega)$

$$H(j\omega) = \frac{2(-j\omega+2)}{(-\omega^2+1)(j\omega+0.5)(j\omega+1)} = \frac{2(-j\omega+2)(-j\omega+1)(-j\omega+0.5)}{(-\omega^2+1)(0.25+\omega^2)(1+\omega^2)}$$

$$= \frac{2(1-\omega^2-2.5j\omega)(1-j\omega)}{\ldots} \Rightarrow U(\omega) = \frac{2(1-3.5\omega^2)}{(-\omega^2+1)(0.25+\omega^2)(1+\omega^2)}$$

Datele minime ptr. trasarea ramurii de pulsații pozitive sunt prezentate în tabelul de mai jos:

ω	0	$\frac{1}{\sqrt{3.5}}$	1	$\sqrt{3.5}$	$+\infty$
$U(\omega)$	8	0	$-\infty$ | $+\infty$	0.533	0_+
$V(\omega)$	0	$-13.0667\cdot\frac{1}{\sqrt{3.5}}$	$-\infty$ | $+\infty$	0	0_-

i) intersecția cu axele:

- $U(\omega)=0 \Rightarrow 3.5\,\omega^2 = 1 \Rightarrow \omega = \frac{1}{\sqrt{3.5}} \Rightarrow V\left(\frac{1}{\sqrt{3.5}}\right) = \dfrac{-2\cdot\frac{1}{\sqrt{3.5}}\left(\frac{1}{3.5}-3.5\right)}{\left(-\frac{1}{3.5}+1\right)\left(0.25+\frac{1}{3.5}\right)\left(1+\frac{1}{3.5}\right)}$

$$V\left(\frac{1}{\sqrt{3.5}}\right) = \frac{\frac{2}{\sqrt{3.5}}\left(1-3.5^2\right)\cdot 3.5^2}{2.5\cdot\frac{7.5}{4}\cdot 4.5} =$$

$$= \frac{2}{\sqrt{3.5}}\cdot\frac{(1-12.25)\cdot 12.25}{2.5\cdot 1.875\cdot 4.5}$$

$$= \frac{2}{\sqrt{3.5}}\cdot\frac{-11.25\cdot 12.25}{11.25\cdot 1.875}$$

$$= -13.0667\cdot\frac{1}{\sqrt{3.5}}$$

- $V(\omega)=0 \Rightarrow \omega\left(\omega^2-3.5\right)=0 \Rightarrow \omega_1 = 0$

$$\omega_{2,3} = \pm\sqrt{3.5} \Rightarrow U(\sqrt{3.5}) = \frac{2\,(1-3.5\cdot 3.5)}{(-3.5+1)(0.25+3.5)(1+3.5)}$$

$$= \frac{2\,(1-12.25)}{-2.5\cdot 3.75\cdot 4.5} = \frac{2}{3.75} = 0.5333$$

ii) asimptota oblică $(\omega \to 1)$

$$\bar{V} = m\cdot\bar{U} + n$$

$$m = \lim_{\omega\to 1}\frac{V(\omega)}{U(\omega)} = \lim_{\omega\to 1}\frac{2\left(-3.5\omega+\omega^3\right)}{2\left(1-3.5\omega^2\right)} = 1$$

$$n = \lim_{\omega\to 1}\left[V(\omega)-m\cdot U(\omega)\right] = \lim_{\omega\to 1}\frac{2\left(-3.5\omega+\omega^2-1+3.5\omega^3\right)}{\left(-\omega^2+1\right)\left(0.25+\omega^2\right)\left(1+\omega^2\right)} =$$

$$\overset{\text{L'Hopital}}{=} \frac{2}{1.25\cdot 2}\cdot\lim_{\omega\to 1}\frac{-3.5+2\omega+10.5\omega^2}{-2\omega}$$

$$= \frac{\cancel{2}}{2.5}\cdot\frac{9}{-\cancel{2}} = -3.6 \Rightarrow \boxed{\bar{V} = \bar{U} - 3.6}$$

b) conturul din jurul polului j (γ_1)

$$s = j + \rho_1 \cdot e^{j\theta}, \quad \rho_1 \to 0, \quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$H(s)\Big|_{s\in\gamma_1} = \frac{2(-j-\cancel{\rho_1 \cdot e^{j\theta}}+2)}{(\cancel{-1}+2j\rho_1 e^{j\theta}+\cancel{\rho_1^2 e^{2j\theta}}+\cancel{1})(j+\cancel{\rho_1 \cdot e^{j\theta}}+0.5)(j+\cancel{\rho_1 \cdot e^{j\theta}}+1)}\Bigg|_{\rho_1\to 0}$$

$$\cong \frac{2(2-j)}{2j\cdot\rho_1\cdot e^{j\theta}(j+0.5)(j+1)} = M\cdot e^{j\varphi} \Rightarrow M = \frac{2\sqrt{5}}{2\rho_1\cdot\sqrt{1.25}\cdot\sqrt{2}}$$

$$M \to \infty \Big|_{\rho_1\to 0}$$

$$\varphi = -\mathrm{arctg}\frac{1}{2} - \frac{\pi}{2} - \theta - \mathrm{arctg}\,2 - \frac{\pi}{4} =$$

$$= -\frac{3\pi}{4} - \left(\mathrm{arctg}\frac{1}{2} + \mathrm{arctg}\,2\right) - \theta$$

$$= -\frac{5\pi}{4} - \theta \in \left[-\frac{3\pi}{4}, -\frac{7\pi}{4}\right]$$

$$\Delta\varphi = \varphi_f - \varphi_i = -\pi$$

c) conturul la infinit

$$\Gamma: s = R\cdot e^{j\delta}, \quad R\to\infty, \quad \delta \in \left[\frac{\pi}{2}, -\frac{\pi}{2}\right]$$

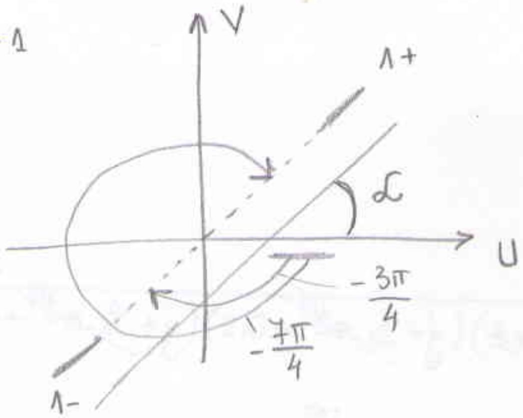
$$H(s)\Big|_{s\in\Gamma} = \frac{2(\cancel{2}-R\cdot e^{j\delta})}{(\cancel{1}+R^2\cdot e^{j2\delta})(\cancel{0.5}+R\cdot e^{j\delta})(\cancel{1}+R\cdot e^{j\delta})} \cong$$

$$\cong \frac{-2}{R^3\cdot e^{j3\delta}} = N\cdot e^{j\psi} \Rightarrow N = \frac{2}{R^3}\Big|_{R\to\infty} \to 0$$

$$\psi = \pi - 3\delta \in \left[-\frac{\pi}{2}, -\frac{5\pi}{2}\right]$$

$$\Delta\psi = \psi_f - \psi_i = 3\pi$$

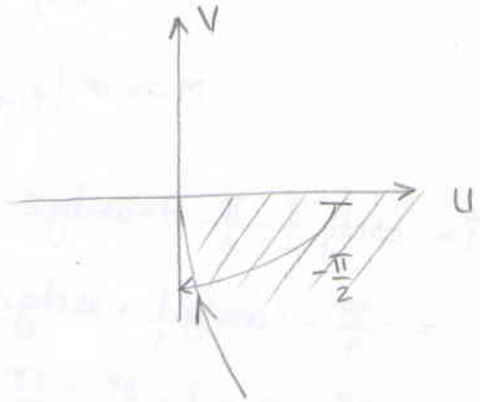
$\omega = 1$



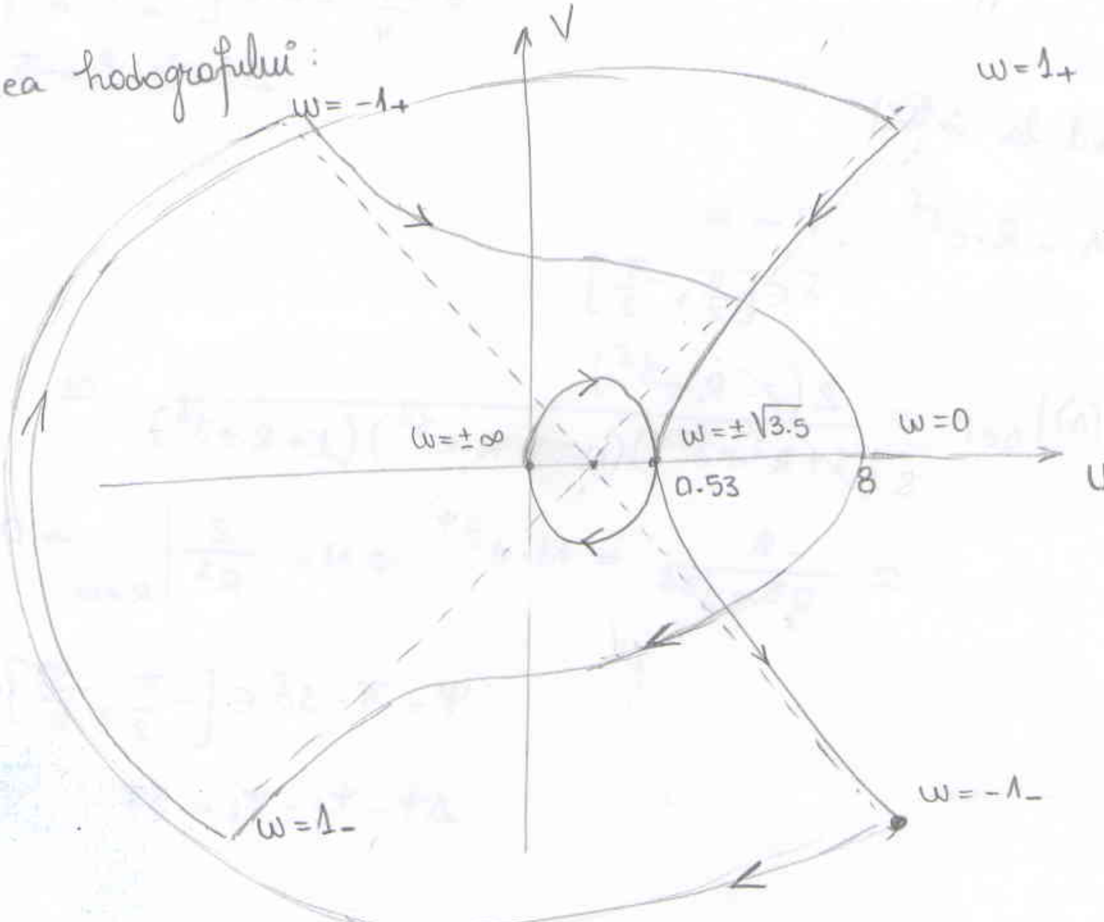
$m = 1$

$\alpha = \frac{\pi}{4}$

$\omega = \infty$



Trasarea hodografului:



Planul $H(s)$

Se trasează inițial hodograful pentru valorile pozitive ale lui ω, apoi se simetrizează.

Zoom în origine: