

AA - Laborator 7

Inductie structurala

Tipul TList

TList := nil | cons(T, TList)

a) Constructori de baza:

nil :-> TList

cons : T x TList -> TList

b) Operatori:

head : TList \ {nil} -> T

tail : TList \ {nil} -> TList

null : TList -> {0,1}

c) Axiome:

head(cons(x,L)) = x

tail(cons(x,L)) = L

null(nil) = 1

null(cons(x,L)) = 0

Algoritmi

1. Concatenarea a doua liste

```
concat(l1,l2){
  if l1 = nil{
    return l2
  }
  else if l1 = cons(x,l3) {
    return cons(x,concat(l3,l2))
  }
}
```

2. Inversarea unei liste

```
rev(l1){
  if l1 = nil{
    return nil
  }
  else if l1 = cons(x,l3){
    return concat(rev(l3),cons(x,nil))
  }
}
```

3. Lungimea unei liste:

```
length(l1){
  if l1 = nil{
    return 0
  }
  else if l1 = cons(x,l3){
    return 1 + length(l3)
  }
}
```

Exercitii

1. Orice l1 din TList, concat(l1,nil) = l1

Rezolvare

Inductie structurala dupa l1:

a) Caz de baza

l1 = nil

concat(l1,nil) = concat(nil,nil) = nil = l1

b) Pas de inductie

Ipoteza de inductie: concat(l1,nil) = l1

Aratam ca: concat(cons(x,l1),nil) = cons(x,l1)

concat(cons(x,l1),nil) = cons(x,concat(l1,nil)) = ip. ind. = cons(x,l1)

2. Orice l1,l2,l3 din TList, concat(l1,concat(l2,l3)) = concat(concat(l1,l2),l3)

Rezolvare

Inductie structurala dupa l1:

a) Caz de baza

l1 = nil

concat(l1,concat(l2,l3)) = concat(nil,concat(l2,l3)) = concat(l2,l3) = concat(concat(nil,l2),l3) =
= concat(concat(l1,l2),l3)

b) Pas de inductie

Ipoteza de inductie: concat(l1,concat(l2,l3)) = concat(concat(l1,l2),l3)

Aratam ca: concat(cons(x,l1),concat(l2,l3)) = concat(concat(cons(x,l1),l2),l3)

concat(cons(x,l1),concat(l2,l3)) = cons(x,concat(l1,concat(l2,l3))) = ip. ind. =

= cons(x,concat(concat(l1,l2),l3)) = concat(cons(x,concat(l1,l2)),l3) =

= concat(concat(cons(x,l1),l2),l3)

3. Orice $l1, l2$ din TList, $rev(concat(l1, l2)) = concat(rev(l2), rev(l1))$

Rezolvare

Inductie structurala dupa l1:

a) Caz de baza

$$l1 = nil$$

$$\begin{aligned} rev(concat(l1, l2)) &= rev(concat(nil, l2)) = rev(l2) = ex1 = concat(rev(l2), nil) = concat(rev(l2), rev(nil)) = \\ &= concat(rev(l2), rev(l1)) \end{aligned}$$

b) Pas de inductie

$$\text{Ipoteza de inductie: } rev(concat(l1, l2)) = concat(rev(l2), rev(l1))$$

$$\text{Aratam ca: } rev(concat(cons(x, l1), l2)) = concat(rev(l2), rev(cons(x, l1)))$$

$$\begin{aligned} rev(concat(cons(x, l1), l2)) &= rev(cons(x, concat(l1, l2))) = concat(rev(concat(l1, l2)), rev(cons(x, nil))) = \\ &= ip. ind. = concat(concat(rev(l2), rev(l1)), rev(cons(x, nil))) = ex2 = \\ &= concat(rev(l2), concat(rev(l1), rev(cons(x, nil)))) = concat(rev(l2), rev(cons(x, l1))) \end{aligned}$$

4. Orice $l1, l2$ din TList, $length(concat(l1, l2)) = length(l1) + length(l2)$

Rezolvare

Inductie structurala dupa l1:

a) Caz de baza

$$l1 = nil$$

$$\begin{aligned} length(concat(l1, l2)) &= length(concat(nil, l2)) = length(l2) = 0 + length(l2) = length(nil) + length(l2) = \\ &= length(l1) + length(l2) \end{aligned}$$

b) Pas de inductie

$$\text{Ipoteza de inductie: } length(concat(l1, l2)) = length(l1) + length(l2)$$

$$\text{Aratam ca: } length(concat(cons(x, l1), l2)) = length(cons(x, l1)) + length(l2)$$

$$\begin{aligned} length(concat(cons(x, l1), l2)) &= length(cons(x, concat(l1, l2))) = 1 + length(concat(l1, l2)) = ip. ind. = \\ &= 1 + length(l1) + length(l2) = length(cons(x, l1)) + length(l2) \end{aligned}$$

5. Orice $l1$ din TList, $length(rev(l1)) = length(l1)$

Rezolvare

Inductie structurala dupa l1:

a) Caz de baza

$$l1 = nil$$

$$length(rev(l1)) = length(rev(nil)) = length(nil) = length(l1)$$

b) Pas de inductie

$$\text{Ipoteza de inductie: } length(rev(l1)) = length(l1)$$

$$\text{Aratam ca: } length(rev(cons(x, l1))) = length(cons(x, l1))$$

$$length(rev(cons(x, l1))) = length(concat(rev(l1), rev(cons(x, nil)))) = ex 4 = length(rev(l1)) +$$

$\text{length}(\text{cons}(x,\text{nil})) = \text{ip. ind.} = \text{length}(l1) + 1 + \text{length}(\text{nil}) = \text{length}(l1) + 1 + 0 = \text{length}(l1) + 1 = \text{length}(\text{cons}(x,l1))$

6. Orice l1 din TList, $\text{rev}(\text{rev}(l1)) = l1$

Rezolvare

Inductie structurala dupa l1:

a) Caz de baza

$l1 = \text{nil}$

$\text{rev}(\text{rev}(l1)) = \text{rev}(\text{rev}(\text{nil})) = \text{rev}(\text{nil}) = \text{nil} = l1$

b) Pas de inductie

Ipoteza de inductie: $\text{rev}(\text{rev}(l1)) = l1$

Aratam ca: $\text{rev}(\text{rev}(\text{cons}(x,l1))) = \text{cons}(x,l1)$

$\text{rev}(\text{rev}(\text{cons}(x,l1))) = \text{rev}(\text{concat}(\text{rev}(l1),\text{cons}(x,\text{nil}))) = \text{ex 3} = \text{concat}(\text{rev}(\text{cons}(x,\text{nil})),\text{rev}(\text{rev}(l1))) =$
 $= \text{ip.ind.} = \text{concat}(\text{rev}(\text{cons}(x,\text{nil})),l1) = \text{concat}(\text{concat}(\text{rev}(\text{nil}), \text{cons}(x,\text{nil})),l1) =$
 $\text{concat}(\text{concat}(\text{nil},\text{cons}(x,\text{nil})),l1) = \text{concat}(\text{cons}(x,\text{nil}),l1) = \text{cons}(x,\text{concat}(\text{nil},l1)) = \text{cons}(x,l1)$

Observatii

- In rezolvarea de fata s-a specificat (pt. egalitati) doar folosirea altor exercitii sau a ipotezei de inductie. Restul egalitatilor se obtin din folosirea celor 3 algoritmi (cititi de la intrari spre iesiri sau invers).

- In rezolvarea problemelor de tipul celor de mai sus (la lucrari sau la examen), este bine sa scrieti deasupra fiecarui egal cum ati obtinut expresia din dreapta egalului (folosind un algoritm, proprietate, ipoteza de inductie, ...).